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Multiple Regression and Serially Correlated Errors

A Monte Carlo Study of the Small Sample Properties of Various Two Stage Estimators.

This paper concerns the estimation of the coefficients of a regression equation of which the error terms are serially correlated. The method of Least Squares and three two-stage estimating methods are examined by means of a Monte Carlo experiment. The methods are appraised on the basis of the sampling properties of the estimates generated by them; taking into account the stochastic variation which is necessarily present in distribution sampling applications.
Two variants of the same model have been used to make it possible to compare the methods in the presence of substantial multicollinearity in the explanatory variables.

1. Introduction

Consider the regression model

$$\underline{y}_t = b_o + b_1 x_{1t} + \ldots\ldots + b_K x_{Kt} + \underline{\varepsilon}_t \qquad t = 1,2,\ldots.T \qquad (1)$$

where:

- $\underline{y}_t$: is the observation of the dependent variable at time t;
- $x_{1t},\ldots,x_{Kt}$: are the observations on the K explanatory variables at time t. These variables are non-stochastic and linear independent;
- $\underline{\varepsilon}_t$: is the error term at time t;
- $T > K + 1$

It is assumed that

- $E(\underline{\varepsilon}_t) = 0$
- $E(\underline{\varepsilon}_t^2) = \sigma^2$

- $E(\underline{\varepsilon}_t\ \underline{\varepsilon}_\theta) = 0 \qquad t \neq \theta$ (3^b)
- the error terms are normally and independently distributed (4)

As is well known, the least squares estimators of b_o through b_K in that case a.o. the following properties:

- They are the best linear unbiased estimators;
- They are normally distributed.

For economic data, however the assumption (3^b) will seldom be a realistic one. On the contrary there are good reasons to assume that the error terms in successive periods are strongly positively autocorrelated, as is shown by Cochrane and Orcutt [1] . If the form of the autoregression structure of the error term is known, estimation methods exist which will produce estimates being generally asymptotically equivalent tot the linear estimate with minimal dispersion. These estimators are complicated expressions of the observations so that, in many cases, it is impossible to determine the exact analytic form of their distributions for finite sample size. However, the asymptotic properties are of little use to econometricians typically working with small samples of data; small sample properties of estimators being in fact of utmost importance.

The only remaining method to obtain a better insight in these small sample properties is simulation. The purpose of this study is, with the aid of simulation, to form an opinion about the merits of the various estimation methods.

2. The error process

There are of course many ways in which the successive errors can be interdependent. Restricting ourselves to economic data, many writers, among whom Christ [2] , Goldberger [4] and Malinvaud [6] suggest that the error process satisfactorily can be described by a first order Markov schema with a coefficient ρ between 0 and 1, which means that, one assumes

$$\underline{\varepsilon}_t = \rho\ \underline{\varepsilon}_{t-1} + \underline{\eta}_t \tag{5}$$

with

- $E(\underline{\eta}_t) = 0$ (6)

- $E(\underline{\eta}_t^2) = \sigma^2$ (7^a)

- $E(\underline{\eta}_t\ \underline{\eta}_\theta) = 0 \qquad t \neq \theta$ (7^b)

- the $\underline{\eta}_t$ are normally and independently distributed (8)

From above it can easily be seen that:

- $E(\underline{\varepsilon}_t) = 0$ (9)

- $E(\underline{\varepsilon}_t^2) = \dfrac{\sigma^2}{1-\rho^2}$ (10^a)

- $E(\underline{\varepsilon}_t\ \underline{\varepsilon}_\theta) = \dfrac{\sigma^2}{1-\rho^2}\ \rho^{|t-\theta|} \qquad t \neq \theta$ (10^b)

- the $\underline{\varepsilon}_t$ are normally distributed (11)

In this study we restrict ourselves to the case that the errors follow a first order Markov scheme with a positive coefficient.

3. The estimation methods

If the errors follow a first order Markov scheme with known ρ if it can easily be understood that least squares regression of $\underline{v}_t$ on u_{it}, where

$$\underline{v}_t = \underline{y}_t - \rho\ \underline{y}_{t-1} \qquad t = 2,3,\ldots..T \tag{12}$$

$$u_{it} = x_{it} - \rho\, x_{it-1} \quad \begin{array}{l} t = 2,3,\ldots\ldots T \\ i = 1,2,\ldots\ldots K \end{array} \tag{13}$$

provides linear estimates, which are unbiased and have minimal dispersion.
For

$$\underline{v}_t = b_o\,(1-\rho) + b_1\, u_{it} + \ldots\ldots + b_K\, u_{Kt} + \underline{\eta}_t \tag{14}$$

and the $\underline{\eta}_t$ are not autocorrelated.
In practice, however, ρ is not known consequently the above described method is not applicable.

It can be proved that if ρ is replaced by an estimated coefficient $\hat{\underline{\rho}}$, the so computed estimates are asymptotically equivalent to the linear estimate with minimal dispersion if $\hat{\underline{\rho}}$ is a consistent estimator of ρ [6] .

As there are various methods to find consistent estimators of ρ, there are also several estimation methods to estimate b_o through b_K.

Three of these methods will be described below.

M 1) This method, first stated by Cochrane and Orcutt [1] and more systematically applied by Klein [5] , while Sargan [8] has shown that it can be generalised to models with several equations. The basic idea of this method is as follows:
The relation (14) can be written in the form

$$\underline{y}_t = \rho\, \underline{y}_{t-1} + b_o(1 - \rho) + b_1\, x_{1t} - \rho\, b_1\, x_{1t-1} + \ldots\ldots$$

$$+ b_K\, x_{Kt} - \rho\, b_K\, x_{Kt-1} + \underline{\eta}_t \tag{15}$$

one can estimate ρ and b_o through b_K simultaneously by the method of Least Squares considering the a priori restrictions on the coefficients. As direct calculation of ρ and b_o through b_K is troublesome, the following iterative procedure is preferred.

a) On the basis of an a priori chosen value $\rho^{(o)}$ of ρ (e.g. $\rho^{(o)} = 0$) the least squares estimators $\underline{\hat{b}}_o^{(1)}$ through $\underline{\hat{b}}_K^{(1)}$ of b_o through b_K can be determined.

b) Then, assuming that $\underline{\hat{b}}_o^{(1)}$ through $\underline{\hat{b}}_K^{(1)}$ is correct, the least squares estimator $\underline{\hat{\rho}}^{(2)}$ of ρ can be calculated.

c) Afterwards one calculates, assuming that $\rho = \underline{\hat{\rho}}^{(2)}$, the least squares estimators $\underline{\hat{b}}_o^{(3)}$ through $\underline{\hat{b}}_K^{(3)}$ of b_o through b_K and so on.

This procedure will be ended as soon as the required accuracy, in this study four significant figures, has been reached.

M 2) An other estimation method developed by Durbin [3a,b] consists of application of least squares on the relation (15) ignoring the a priori restrictions. The so obtained estimator $\underline{\hat{\rho}}$ is used to compute, assuming that $\rho = \underline{\hat{\rho}}$, the least squares estimators of b_o through b_K.

The asymptotic efficiency of this method is the same as that of the first method described above.

M 3) This method proceeds from (15). The first steps in the calculation are the same as a) and b) of the method described under M 1, provided $\rho^{(o)} = 0$. Malinvaud [6] has shown that a practical unbiased estimate of ρ is obtained by taking

$$\left(1 + \frac{K + 1}{T - K - 1}\right) \underline{\hat{\rho}}^{(2)} \qquad (16)$$

as estimator if the explanatory variables have very irregular evolutions, and

$$\underline{\hat{\rho}}^{(2)} + \frac{K + 1}{T} (1 + \underline{\hat{\rho}}^{(2)}) \qquad (17)$$

if these variables have, as he mentions, smooth evolutions.

He suggests to take

$$\underline{\hat{\rho}}^{(2)} + \frac{K + 1}{T} \qquad (18)$$

as an estimator of ρ and with the aid of (15), to calculate the least squares estimators of b_o through b_K.

4. The experiment.

The basic model to be considered here is given by the equations

$$\underline{y}_t = b_o + b_1 x_{1t} + b_2 x_{2t} + \underline{\varepsilon}_t \tag{19}$$

where

$$b_o = 0 \quad , \quad b_1 = 1 \quad , \quad b_2 = 1$$

and

$$\underline{\varepsilon}_t = \rho\, \underline{\varepsilon}_{t-1} + \underline{\eta}_t \tag{20}$$

where

- $E(\underline{\eta}_t) = 0 \qquad t = 1,2,.....T$
- $E(\underline{\eta}_t^2) = 5 \qquad t = 1,2,.....T$
- $E(\underline{\eta}_t\ \underline{\eta}_\theta) = 0 \qquad t, \theta = 1,2,.....T \quad t \neq \theta$
- the $\underline{\eta}_t$ are normally and independently distributed

Using a random sampling method 100 series of 40 ε-values are simulated for different values of ρ viz. $\rho = 0.0$, $\rho = 0.4$ and $\rho = 0.8$.

Each series was used twice, one in an A and one in a B experiment. The A experiments differ from the B ones in the extent to with the explanatory variables were specified to be intercorrelated.

The explanatory variables of model A are not intercorrelated; the ones of model B are heavily intercorrelated (see table I). This to get an insight in the effect of multicollinearity on the estimation methods. With the help of table I and (19) for every series of $\underline{\varepsilon}_t$'s the values of the dependent variable $\underline{y}_t$

are computed.
Finally the method of least squares and each of the described estimation methods in 3. were applied to every series of 40 (y_t, x_{1t}, x_{2t}) values to obtain.

a) estimates of ρ, b_o, b_1 and b_2 ;

b) estimates of the variances of the different estimators of b_o, b_1 and b_2.

The same calculations have been carried out starting with the first 25, respectively 15 values of every series; so that, for model A as well as model B one has for every combination of ρ and T (ρ = 0.0; 0.4; 0.8 and T = 15; 25; 40), 100 x 4 estimates of ρ, b_o, b_1 and b_2 at his disposal.

5. The analysis

In order to give a judgement on the various estimation methods one needs a measure of dispersion. The two measures most commonly used are the Mean Square Error (M.S.E.) and the Mean Absolute Error (M.A.E.).
The advantage of the Mean Square Error is that it is a simple function of the bias and variance of the frequency function. The Mean Absolute Error figures importantly because it is simple to make certain statistical test based upon it.
In the tables $2^{A,B}$ through $5^{A,B}$ the bias, variance and the M.S.E. of the various parameters are given.
In the tables 6^A and 6^B the estimation methods are ranked with the aid of the M.S.E. criterion. The ranking achieved in this way has the advantage that it is impossible to judge which observed differences are in fact statistically significant.
It appears from the tables $2^{A,B}$ through $4^{A,B}$ that the bias of b_o, b_1 and b_2 can be neglected, thus the M.S.E. is equalling the variance. As is also known the estimators of these coefficients are asymptotically normally distributed so that the Pitman test [7] can be applied by approximation and

this makes pairwise comparison of the various methods possible.
In the tables 7^A and 7^B the outcomes of a χ^2 test on normality are presented, the ones of the Pitman test in the tables 8^A en 8^B.
With the help of these outcomes an attempt is made to come to a ranking of the various methods (see tables 9^A and 9^B).
When ranking the estimation methods on the basis of this procedure several difficulties will be encountered:

1° Ranking of the estimation methods with relation to the coefficient ρ is impossible;

2° If the χ^2 test on normality leads to rejection no ranking is possible;

3° The pairwise comparisons may display intransitivity so that a consistent ranking cannot be achieved.

To start with the last mentioned difficulty no intransitivities were actually observed. The other two difficulties can be avoided if one passes to pairwise comparison of the M.A.E. for which Summers 9 has designed a non parametric test.
In the tables 10^A and 10^B the outcomes of this test are presented, while in the tables 11^A and 11^B an attempt is made to come to a ranking of the several estimation methods (with relation to b_o, b_1, b_2 and ρ).
This time several intransitivities were observed but fortunately not in the important cases that $\rho = 0.4$ or $\rho = 0.8$.

It is essential that an estimate $\hat{\underline{b}}$ of b is accompanied by a measure of its precision. It is easy to compute for each of the methods an estimate of the standard error $\hat{\underline{\sigma}}_b$ of the coefficient b but how good is this estimate for small samples?
Can judgements about a coefficient be made on the basis of $\hat{\underline{b}}$ and $\hat{\underline{\sigma}}_b$?
To answer these questions a series of χ^2 tests were performed to see if

$$\frac{\hat{\underline{b}} - b}{\hat{\underline{\sigma}}_b}$$

were by approximation t distributed.
Table 12^A and 12^B presents the results of these tests.

6. The results.

In order to be able to reach a conclusion it is necessary to introduce a scoring system. The system adopted in this study is the following.
A score is assigned to each method in each column of the tables $9^{A,B}$ and $11^{A,B}$. This score has been found by comparison of the method with all the methods in the same column to which it was superior. The points awarded to a method in a particular comparison depended on the size of the entry in the tables $8^{A,B}$ and $10^{A,B}$ corresponding to the comparison. The size of an entry is monotonically related to the probability that the method is really superior so that the points are awarded according to the size of the entry.
These scores were for entries

between 0.00 and 0.45	1 point
0.50 and 0.99	2 points
1.00 and 1.49	3 points
1.50 and 1.96	4 points
1.96 and ∞	6 points

Both for the ranking with the aid of the Pitman test and the ranking with the aid of the Summers test an average score is computed for every combination of ρ and T.
These average scores are presented in tables 13 and 14.
Although there are some slight differences in the ranking of the methods with the aid of tables 13 and 14 it is possible to get an idea about the merits of the estimation methods in the different cases.
For both rankings for model A as well as for model B the conclusions are the following:

1) The least squares method is the best for $\rho = 0.0$ and $T = 15$ or 25 as can be proven theoretically;

2) The method M 3 is the best for $\rho = 0.4$ or $\rho = 0.8$ and $T = 15$ or 25;

3) The least squares method is the worst for $\rho = 0.4$ or $= 0.8$ and $T = 15$, 25 or 40;

4) For the case $\rho = 0.4$ or 0.8 and $T = 40$ no method is demonstrable as the best, probably because they are asymptotically equivalent;

5) For the case $\rho = 0.0$ and $T = 40$ no method is demonstrable as the best. In all probability because the methods M 1, M 2 and M 3 are asymptotically equivalent with least squares;

6) There is no demonstrable difference for the various methods between model A and model B.

Regarding the accuracy of the estimates there is a difference between model A and model B. From table 12^A and 12^B it appears that for model A 7% of the null hypotheses was rejected, while this amounted to 14% for model B.

7. Acknowledgements.

I want to thank Mr. J. Pompen for programming the simulation problem described in section 4.

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9. Tables.

Table 1.	MODEL A						MODEL B					
		x_1	x_2		x_1	x_2		x_1	x_2		x_1	x_2
	1	51.01	40.94	21	23.93	44.18	1	51.01	43.18	21	23.93	22.81
	2	57.77	43.71	22	44.10	28.79	2	57.77	49.70	22	44.10	32.80
	3	27.64	41.06	23	37.22	52.82	3	27.64	24.54	23	37.22	36.90
	4	46.06	44.65	24	42.52	36.15	4	46.06	40.71	24	42.52	34.48
	5	35.51	40.79	25	41.67	42.49	5	35.51	30.73	25	41.67	36.33
	6	47.25	43.59	26	27.26	32.35	6	47.25	41.24	26	27.26	20.75
	7	42.90	41.11	27	49.66	34.25	7	42.90	36.77	27	49.66	39.43
	8	45.10	45.00	28	48.37	39.40	8	45.10	40.08	28	48.37	40.46
	9	32.73	40.24	29	38.22	30.10	9	32.73	28.28	29	38.22	28.62
	10	59.79	26.92	30	43.86	37.56	10	59.79	44.60	30	43.86	36.11
	11	39.97	44.53	31	52.63	32.98	11	39.97	35.79	31	52.63	41.30
	12	53.18	44.18	32	46.46	27.01	12	53.18	46.21	32	46.46	33.98
	13	34.25	38.07	33	35.57	45.11	13	34.25	28.63	33	35.57	32.50
	14	46.15	38.86	34	54.92	16.01	14	46.15	38.46	34	54.92	36.34
	15	43.47	27.32	35	36.63	35.79	15	43.47	31.71	35	36.63	29.62
	16	38.15	37.89	36	26.00	21.79	16	38.15	31.67	36	26.00	15.52
	17	41.59	59.11	37	24.56	32.57	17	41.59	42.91	37	24.56	18.68
	18	40.51	38.69	38	54.31	38.24	18	40.51	33.89	38	54.31	44.74
	19	37.43	42.11	39	34.27	46.24	19	37.43	32.79	39	34.27	31.92
	20	34.15	41.89	40	41.09	43.94	20	34.15	30.08	40	41.09	36.45

Table 2[A] The Mean Biases, Variances and Mean-Square Errors of the Coefficient b_o.

		T = 15			T = 25			T = 40		
b_o		ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80
TRUE VALUE		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
L S	BIAS	-0.2178	0.0631	1.0938	-1.4691	-0.9016	0.1348	-0.5962	-0.3956	-0.3272
	VAR	152.30	162.85	303.31	85.17	82.46	134.31	24.91	34.75	116.65
	MSE	152.34	162.86	304.50	87.33	83.27	134.33	25.26	34.91	116.76
M I	BIAS	-0.8055	-0.2273	0.8136	-1.6073	-1.0169	-0.4558	-0.4279	-0.3176	-0.2757
	VAR	218.32	160.89	182.36	96.58	67.40	55.55	24.26	22.54	27.70
	MSE	218.97	160.94	183.02	99.17	68.43	55.76	24.44	22.64	27.78
M 2	BIAS	-0.8291	-0.4516	2.5262	-1.4904	-0.5344	-0.7394	-0.4525	-0.3484	-0.2675
	VAR	213.96	167.72	512.62	94.08	107.96	69.39	24.38	23.01	27.72
	MSE	214.65	167.93	519.00	96.30	108.25	69.93	24.59	23.13	27.79
M 3	BIAS	-0.3449	-0.1188	0.5092	-1.4636	-0.9941	-0.3335	-0.4688	-0.3360	-0.2617
	VAR	173.15	135.07	150.29	93.13	66.92	68.61	24.56	22.74	27.79
	MSE	173.27	135.09	150.55	95.27	67.91	58.73	24.78	22.85	27.86

Table 2[B] The Mean Biases, Variances and Mean Square Errors of the Coefficient b_o.

		T = 15			T = 25			T = 40		
b_o		ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80
TRUE VALUE		0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
L S	BIAS	-0.4054	-0.0716	0.5009	-0.6646	-0.3445	-0.4018	-0.1882	0.0286	0.2174
	VAR	56.64	46.82	60.76	33.16	30.69	62.18	12.64	14.45	37.69
	MSE	56.80	46.83	61.01	33.61	30.81	62.34	12.68	14.45	37.74
M I	BIAS	-0.4934	0.0225	0.8915	-0.7992	-0.3369	0.0099	-0.1447	0.0562	0.1101
	VAR	76.67	53.17	145.49	37.89	27.51	34.45	12.57	11.57	20.30
	MSE	76.91	53.17	146.29	38.53	27.62	34.45	12.59	11.57	20.31
M 2	BIAS	-0.5653	-0.3119	2.5868	-0.7806	0.0820	-0.2974	-0.1501	0.0392	0.1183
	VAR	67.90	51.01	361.11	36.35	62.03	49.90	12.52	11.55	20.15
	MSE	68.22	51.11	367.80	36.96	62.04	49.99	12.55	11.56	20.16
M 3	BIAS	-0.3277	-0.1416	0.4783	-0.6587	-0.3325	0.1333	0.1395	0.0535	0.1282
	VAR	59.62	41.49	71.11	37.00	27.20	40.23	12.65	11.65	20.31
	MSE	59.73	41.51	71.34	37.43	27.31	40.25	12.67	11.65	20.32

Table 3[A] The Mean Biases, Variances and Mean Square Errors of the Coefficient b_1

	T = 15			T = 25			T = 40		
b_1	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80
TREU VALUE	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
L S BIAS	0.0127	0.0051	-0.0084	0.0140	0.0073	-0.0094	0.0039	-0.0012	0.0067
L S VAR	0.0266	0.0196	0.0169	0.0164	0.0138	0.0047	0.0066	0.0066	0.0132
L S MSE	0.0267	0.0196	0.0169	0.0166	0.0139	0.0248	0.0067	0.0066	0.0132
M I BIAS	0.0132	0.0071	0.0055	0.0166	0.0060	0.0030	0.0029	-0.0020	-0.0039
M I VAR	0.0356	0.0199	0.0131	0.0188	0.0124	0.0090	0.0067	0.0052	0.0046
M I MSE	0.0357	0.0199	0.0132	0.0191	0.0125	0.0090	0.0067	0.0052	0.0046
M 2 BIAS	0.0131	0.0076	0.0045	0.0166	0.0074	0.0031	0.0030	-0.0016	0.0034
M 2 VAR	0.0316	0.0198	0.0128	0.0179	0.0121	0.0089	0.0067	0.0052	0.0046
M 2 MSE	0.0318	0.0199	0.0128	0.0182	0.0122	0.0089	0.0067	0.0052	0.0046
M 3 BIAS	0.0100	0.0076	0.0046	0.0132	0.0061	0.0020	0.0026	-0.0020	-0.0036
M 3 VAR	0.0282	0.0173	0.0120	0.0184	0.0121	0.0090	0.0067	0.0052	0.0046
M 3 MSE	0.0283	0.0173	0.0120	0.0186	0.0122	0.0090	0.0068	0.0052	0.0046

Table 3^B

The Mean Biases, Variances and Mean-Square Errors of the Coefficient b_1

		T = 15			T = 25			T = 40		
b_1		ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.080
TRUE VALUE		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
L S	BIAS	0.0234	0.0128	0.0255	-0.0319	0.0245	-0.0216	-0.0195	-0.0255	-0.0380
	VAR	0.2267	0.2596	0.5094	0.1182	0.1018	0.1033	0.0542	0.0573	0.1319
	MSE	0.2272	0.2598	0.5101	0.1192	0.1024	0.1030	0.0546	0.0580	0.1333
M I	BIAS	-0.0046	-0.0071	0.0010	-0.0296	-0.0328	-0.0236	-0.0133	-0.0233	-0.0259
	VAR	0.2842	0.3020	0.4064	0.1278	0.0989	0.0787	0.0564	0.0479	0.0387
	MSE	0.2842	0.3021	0.4064	0.1287	0.0999	0.0792	0.0566	0.0484	0.0394
M 2	BIAS	-0.0020	0.0004	0.0006	-0.0240	-0.0279	-0.0221	-0.0143	-0.0237	-0.0254
	VAR	0.3018	0.3064	0.4208	0.1281	0.1025	0.0796	0.0564	0.0479	0.0389
	MSE	0.3018	0.3064	0.4208	0.1287	0.1033	0.0800	0.0566	0.0485	0.0395
M 3	BIAS	0.0091	0.0089	0.0064	-0.0328	-0.0317	-0.0247	-0.0163	-0.0243	-0.0259
	VAR	0.2764	0.2785	0.3764	0.1262	0.0981	0.0783	0.0578	0.0474	0.0393
	MSE	0.2765	0.2786	0.3765	0.1273	0.0991	0.0789	0.0581	0.0480	0.0400

Table 4[A]

The Mean Biases, Variances and Mean-Square Errors of the Coefficient b_2.

		T = 15			T = 25			T = 40		
b_2		ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80
TRUE VALUE		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
L S	BIAS	-0.0054	-0.0038	-0.0170	0.0230	0.0159	0.0076	0.0117	0.0121	0.0156
	VAR	0.0537	0.0649	0.1362	0.0277	0.0255	0.0282	0.0108	0.0132	0.0373
	MSE	0.0537	0.0650	0.1365	0.0282	0.0257	0.0282	0.0110	0.0134	0.0375
M I	BIAS	0.0089	0.0071	0.0022	0.0231	0.0194	0.0133	0.0081	0.0107	0.0110
	VAR	0.0721	0.0735	0.0926	0.0304	0.0228	0.0167	0.0111	0.0099	0.0078
	MSE	0.0722	0.0735	0.0962	0.0309	0.0231	0.0169	0.0112	0.0100	0.0080
M 2	BIAS	0.0075	0.0040	0.0020	0.0203	0.0177	0.0126	0.0086	0.0111	0.0110
	VAR	0.0758	0.0744	0.0954	0.0303	0.0238	0.0169	0.0111	0.0100	0.0079
	MSE	0.0759	0.0744	0.0954	0.0307	0.0242	0.0171	0.0112	0.0101	0.0080
M 3	BIAS	0.0005	-0.0006	-0.0009	0.0230	0.0189	0.0133	0.0094	0.0111	0.0111
	VAR	0.0662	0.0649	0.0036	0.0296	0.0227	0.0166	0.0114	0.0098	0.0080
	MSE	0.0662	0.0649	0.0836	0.0302	0.0230	0.0168	0.0115	0.0100	0.0081

Table 4[B]

The Mean Biases, Variances and Mean Square Errors of the Coefficient b_2.

		T = 15			T = 25			T = 40		
b_2		ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80
TRUE VALUE		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
L S	BIAS	-0.0134	-0.0097	-0.0424	0.0574	0.0397	0.0190	0.0293	0.0304	0.0390
	ZAR	0.3354	0.4059	0.8512	0.1730	0.1592	0.1761	0.0678	0.0826	0.2331
	MSE	0.3356	0.4060	0.8530	0.1763	0.1608	0.1764	0.0687	0.0835	0.2347
M I	BIAS	0.0222	0.0178	0.0056	0.0578	0.0486	0.0333	0.0203	0.0267	0.0275
	VAR	0.4509	0.4591	0.5790	0.1899	0.1423	0.1044	0.0695	0.0617	0.0490
	MSE	0.4514	0.4594	0.5790	0.1932	0.1446	0.1055	0.0699	0.0624	0.0498
M 2	BIAS	0.0188	0.0100	0.0049	0.0507	0.0441	0.0316	0.0217	0.0277	0.0276
	VAR	0.4739	0.4649	0.4964	0.1896	0.1491	0.1056	0.0697	0.0624	0.0495
	MSE	0.4742	0.4650	0.5964	0.1922	0.1510	0.1066	0.0701	0.0632	0.0503
M 2	BIAS	0.0012	0.0016	-0.0022	0.0575	0.0472	0.0334	0.0236	0.0278	0.0278
	VAR	0.4135	0.4057	0.5224	0.1853	0.1417	0.1039	0.0713	0.0614	0.0500
	MSE	0.4135	0.4057	0.5224	0.1886	0.1440	0.1050	0.0718	0.0622	0.0507

Table 5[A]

The Mean Biases, Variances and Mean-Square Errors of the Coefficient ρ

		T = 15			T = 25			T = 40		
		$\rho = 0.00$	$\rho = 0.40$	$\rho = 0.80$	$\rho = 0.00$	$\rho = 0.40$	$\rho = 0.80$	$\rho = 0.00$	$\rho = 0.40$	$\rho = 0.80$
TRUE VALUE		0.0000	0.4000	0.8000	0.0000	0.4000	0.8000	0.0000	0.4000	0.8000
L S	BIAS	0.0000	-0.4000	-0.8000	0.0000	-0.4000	-0.8000	0.0000	-0.4000	-0.8000
	VAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	MSE	0.0000	0.1600	0.6400	0.0000	0.1600	0.6400	0.0000	0.1600	0.6400
M I	BIAS	0.0167	-0.0964	-0.2259	0.0076	-0.0841	-0.1882	-0.0067	-0.0744	-0.1401
	VaR	0.0817	0.0771	0.0742	0.0432	0.0366	0.0237	0.0294	0.0230	0.0125
	MSE	0.0819	0.0864	0.1252	0.0433	0.0436	0.0591	0.0295	0.0286	0.0321
M 2	BIAS	0.0736	-0.0300	-0.1520	0.0091	-0.0732	-0.1543	-0.0048	-0.0752	-0.1316
	VAR	0.0905	0.0913	0.0785	0.0463	0.0452	0.0321	0.0280	0.0231	0.0139
	MSE	0.0959	0.0922	0.1016	0.0463	0.0505	0.0599	0.0280	0.0288	0.0312
M 3	BIAS	0.2138	0.0518	-0.1251	0.1214	-0.0018	-0.1256	0.0675	-0.0283	-0.1207
	VAR	0.0451	0.0459	0.0443	0.0337	0.0301	0.0208	0.0244	0.0194	0.0122
	MSE	0.0908	0.0486	0.0599	0.0484	0.0301	0.0366	0.0296	0.0202	0.0268

Table 5[B]

The Mean Biases, Variances and Mean Square Errors of the Coefficient ρ

		T = 15			T = 25			T = 40		
		ρ = 0.00	ρ = 0.40	ρ= 0.80	ρ = 0.00	ρ = 0.40	ρ = 0.80	ρ = 0.00	ρ = 040	ρ = 0.80
TRUE VALUE		0.0000	0.4000	0.8000	0.0000	0.4000	0.8000	0.0000	0.4000	0.8000
L S	BIAS	0.0000	-0.4000	-0.8000	0.0000	-0.4000	-0.8000	0.0000	-0.4000	-0.8000
	VAR	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	MSE	0.0000	0.1600	0.6400	0.0000	0.1600	0.6400	0.0000	0.1600	0.6400
M I	BIAS	0.0167	-0.0964	-0.2259	0.0076	-0.0841	-0.1881	-0.0067	-0.0744	-0.1401
	VAR	0.0817	0.0771	0.0742	0.0432	0.0369	0.0237	0.0294	0.0230	0.0125
	MSE	0.0819	0.0864	0.1252	0.0433	0.0440	0.0591	0.0295	0.0286	0.0321
M 2	BIAS	0.0736	-0.0310	-0.1520	0.0091	-0.0732	-0.1543	-0.0049	-0.0752	-0.1316
	VAR	0.0904	0.0913	0.0785	0.0463	0.0452	0.0321	0.0280	0.0231	0.0139
	MSE	0.0959	0.0923	0.1016	0.0463	0.0505	0.0559	0.0280	0.0288	0.0312
M 3	BIAS	0.2138	0.0518	-0.1251	0.1214	-0.0017	-0.1256	0.0675	-0.0283	-0.1207
	VAR	0.0451	0.0459	0.0443	0.0337	0.0301	0.0208	0.0244	0.0194	0.0122
	MSE	0.0908	0.0486	0.0599	0.0484	0.0301	0.0366	0.0288	0.0202	0.0268

Table 6[A] The Ranking of LS, M_1, M_2, and M_3 with the Aid of the M.S.E. criterion.

$\rho = 0.0$	b_o	b_1	b_2	ρ	$\rho = 0.4$	b_o	b_1	b_2	ρ
T = 15	LS M_3 M_2 M_1	LS M_3 M_2 M_1	LS M_3 M_1 M_2	LS M_1 M_3 M_2	T = 15	M_3 M_1 LS M_2	M_3 LS M_1 M_2	M_3 LS M_1 M_2	M_3 M_1 M_2 LS
T = 25	LS M_3 M_2 M_1	LS M_2 M_3 M_1	LS M_3 M_2 M_1	LS M_1 M_2 M_3	T = 25	M_3 M_1 LS M_2	M_2 M_3 M_1 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS
T = 40	M_1 M_2 M_3 LS	LS M_1 M_2 M_3	LS M_1 M_2 M_3	LS M_2 M_1 M_3	T = 40	M_1 M_3 M_2 LS	M_1 M_2 M_3 LS	M_1 M_3 M_2 LS	M_3 M_1 M_2 LS

Table 6^A

$\rho = 0.8$	b_o	b_1	b_2	ρ
T = 15	M_3 M_1 LS M_2	M_3 M_2 M_1 LS	M_3 M_1 M_2 LS	M_3 M_2 M_1 LS
T = 25	M_1 M_3 M_2 LS	M_2 M_1 M_3 LS	M_3 M_1 M_2 LS	M_3 M_2 M_1 LS
T = 40	M_1 M_2 M_3 LS	M_1 M_2 M_3 LS	M_1 M_2 M_3 LS	M_3 M_2 M_1 LS

Methods with the same M.S.E. are indicated by accolades.

Table 6^B The Ranking of LS, M_1, M_2, and M_3 with the Aid of the M.S.E. criterion.

$\rho = 0.0$	b_o	b_1	b_2	ρ	$\rho = 0.4$	b_o	b_1	b_2	ρ
T = 15	LS M_3 M_2 M_1	LS M_3 M_1 M_2	LS M_3 M_1 M_2	LS M_1 M_3 M_2	T =15	M_3 LS M_2 M_1	LS M_3 M_1 M_2	M_3 LS M_1 M_2	M_3 M_1 M_2 LS
T = 25	LS M_2 M_3 M_1	LS M_3 M_1 M_2	LS M_3 M_2 M_1	LS M_1 M_2 M_3	T = 25	M_3 M_1 LS M_2	M_3 M_1 LS M_2	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS
T = 40	M_2 M_1 M_3 LS	LS M_1 M_2 M_3	LS M_1 M_2 M_3	LS M_2 M_3 M_1	T = 40	M_2 M_1 M_3 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS

Table 6^B

$\rho = 0.08$	b_o	b_1	b_2	ρ
T = 15	LS M_3 M_1 M_2	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS	M_3 M_2 M_1 LS
T = 25	M_1 M_3 M_2 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS	M_3 M_2 M_1 LS
T = 40	M_2 M_1 M_3 LS	M_1 M_2 M_3 LS	M_1 M_2 M_3 LS	M_3 M_2 M_1 LS

Methods with the same M.S.E. are indicated by accolades.

Table 7^A

Results of Applying the X^2 Test 10 df. to the Sample Distributions of the Normalized Coefficients b_o, b_1 and b_2.

		T = 15			T = 25			T = 40		
		b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
L S	ρ = 0.0	6.3	7.3	3.2	5.7	11.1	6.6	3.9	13.7	19.6+
	ρ = 0.4	10.5	12.3	7.9	10.0	9.9	7.4	7.4	18.0+	13.9
	ρ = 0.8	6.4	10.3	4.5	6.9	1.8	3.5	5.1	11.4	4.5
M 1	ρ = 0.0	7.1	7.8	14.2	6.8	10.7	14.4	6.7	8.9	10.7
	ρ = 0.4	8.1	4.4	4.0	5.2	3.2	9.8	6.6	9.7	8.9
	ρ = 0.8	15.1	7.2	14.1	10.1	5.7	6.4	13.8	8.8	5.9
M 2	ρ = 0.0	10.7	11.8	11.5	5.2	11.2	12.4	10.0	10.6	12.4
	ρ = 0.4	13.9	7.2	6.6	9.9	9.8	9.0	7.2	7.3	8.3
	ρ = 0.8	54,9+	5.0	16.2+	8.3	5.9	6.0	16.1+	9.3	4.3
M 3	ρ = 0.0	7.9	9.1	4.9	8.0	3.2	9.5	4.1	13.9	14.8
	ρ = 0.4	8.0	3.8	3.8	7.1	8.6	6.5	16.0+	9.0	9.7
	ρ = 0.8	9.0	7.9	11.2	7.5	5.7	5.4	9.3	10.8	5.3

A table entry represents an outcome of the test statistic asterisks indicate cases where differences are significant at the 95% level.

Table 7^B Results of Applying the X^2 Test 10 df. to the Sample Distributions of the Normalized Coefficient b_o, b_1, and b_2.

		T = 15			T = 25			T = 40		
		b_o	b_1	b_2	b_o	b_1	b_2	b_0	b_1	b_2
L S	ρ = 0.0	7.1	3.8	3.2	12.7	7.1	6.4	9.3	13.6	19.6^+
	ρ = 0.4	10.3	6.6	9.1	8.5	10.6	5.7	18.3^+	11.7	15.3
	ρ = 0.8	3.7	5.5	3.5	10.2	6.8	3.5	10.9	6.0	5.1
M 1	ρ = 0.0	11.5	9.0	14.2	10.2	9.8	14.0	7.2	13.9	12.4
	ρ = 0.4	10.0	16.0^+	6.7	12.8	9.8	9.4	10.6	6.4	8.9
	ρ = 0.8	16.3^+	12.3	12.7	6.6	8.3	4.9	9.4	8.7	5.9
M 2	ρ = 0.0	5.4	7.8	11.5	12.5	8.4	12.4	6.4	15.1	11.8
	ρ = 0.4	13.6	12.0	6.6	30.8^+	14.5	10.3	11.0	8.6	10.3
	ρ = 0.8	69.4^+	9.5	15.0	8.1	12.4	4.7	14.7	10.9	3.7
M 3	ρ = 0.0	6.3	15.2	6.5	11.9	7.5	11.1	4.7	9.3	14.8
	ρ = 0.4	4.7	9.3	3.8	16.6^+	7.6	5.9	5.1	9.1	9.7
	ρ = 0.8	5.1	13.2	6.9	9.5	8.3	4.4	16.8^+	7.4	5.3

A table entry represents an outcome of the test statistic Asteriks indicate case where differences are significant at the 95% level.

Table 8^A Results of Pairwise Comparison with the Aid of the Pitman Test.

	$\rho = 0.0$			$\rho = 0.4$			$\rho = 0.8$		
T = 15	b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
LS - M_1	-4.64^+	-3.41^+	-4.67^+	0.12	-0.17	-1.09	4.62^+	2.07^+	2.07^+
LS - M_2	-4.44^+	-2.48	-5.37^+	-0.27	-0.14	-1.17		2.33^+	
LS - M_3	-2.06^+	-1.08	-3.43^+	1.84	1.40	0.00	5.15^+	2.90^+	2.65^+
M_1 - M_2	0.63	3.24^+	2.19^+	-0.88	0.19	-0.60		2.11^+	
M_1 - M_3	3.74^+	2.93^+	2.88^+	3.47^+	2.77^+	3.87^+	2.02^+	2.87^+	3.87^+
M_2 - M_3	3.89^+	1.97^+	3.31^+	4.87^+	3.16^+	4.53^+		2.73^+	
T = 25	b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
LS - M_1	-2.32^+	-2.23^+	-2.14^+	-2.27^+	1.02	1.42	6.70^+	6.19^+	3.69^+
LS - M_2	-2.14^+	-1.58	-2.45^+	-2.14^+	1.32	0.89	4.82^+	6.31^+	3.63^+
LS - M_3	-1.78	-1.90	-1.70	2.27^+	1.19	1.41	6.32^+	6.17^+	3.69^+
M_1 - M_2	1.47	2.40^+	0.10	-4.66^+	1.57	-3.16^+	-3.50	1.41	-1.74
M_1 - M_3	1.22	0.72	0.97	0.43	1.02	0.32	-1.53	0.24	0.98
M_2 - M_3	0.33	-0.77	0.85	4.75^+	-0.14	2.90^+	2.92^+	-1.10	2.19^+
T = 40	b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
LS - M_1	0.44	-0.18		3.85^+		2.84^+	10.97^+	6.44^+	10.61^+
LS - M_2	0.38	-0.08		3.80^+		2.85^+		6.50^+	10.64^+
LS - M_3	0.23	-0.29				2.84^+	10.82^+	6.45^+	10.63^+
M_1 - M_2	-0.73	0.99	-0.73	-2.08^+	0.17	-1.26		1.62	-0.79
M_1 - M_3	-0.54	-0.24	-0.54		-0.53	0.36	-0.18	-0.60	-1.59
M_2 - M_3	-0.32	-0.49	-0.32		-0.55	1.26		-1.69	-0.95

A table entry represents an outcome of the test statistic Asteriks indicate cases where diffirences are significant at the 95% level.

Table 8^B Results of Pairwise Comparisons with the Aid of the Pitman Test.

	$\rho = 0.0$			$\rho = 0.4$			$\rho = 0.8$		
T = 15	b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
LS - M_1	-3.66^+	-3.96^+	-4.67^+	-1.46		-1.09		1.22	2.07^+
LS - M_2	-2.49^+	-5.00^+	-5.37^+	-1.02	-1.52	-1.17		1.02	1.89
LS - M_3	-1.05	-3.64^+	-3.43^+	1.75	-0.64	0.00	-1.46	1.65	2.65^+
M_1 - M_2	3.12^+	-2.19^+	-2.19^+	0.53		-0.60		-1.70	-1.30
M_1 - M_3	3.29^+	0.70	2.08^+	3.80^+		3.87^+		3.45^+	3.87^+
M_2 - M_3	2.16^+	2.45^+	3.30^+	3.42^+	3.80	4.53^+		4.53^+	4.87^+
T = 25	b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
LS - M_1	-2.42^+	-2.07^+	-2.14^+	1.17	0.39	1.42	4.71^+	2.07^+	3.69^+
LS - M_2	-1.89	-2.38^+	-2.45^+		-0.09	0.89	3.36^+	1.98^+	3.63^+
LS - M_3	-2.09^+	-1.81	-1.70		0.47	1.41	1.61	2.07^+	3.69^+
M_1 - M_2	2.13^+	-0.18	0.10		-2.40^+	-3.17^+	-5.05^+	-1.67	-1.74
M_1 - M_3	0.80	0.55	0.98		0.61	0.32	-3.80^+	0.85	0.98
M_2 - M_3	-0.54	0.56	0.85		2.51^+	2.90^+	3.15^+	2.06^+	2.20^+
T = 40	b_o	b_1	b_2	b_0	b_1	b_2	b_o	b_1	b_2
LS - M_1	0.10	-0.97			1.97^+	2.84^+	5.24	8.38^+	10.61^+
LS - M_2	0.19	-1.02			2.05^+	2.85^+	5.32	8.45^+	10.64^+
LS - M_3	-0.03	-1.73			2.04^+	2.84^+		8.41^+	10.63^+
M_1 - M_2	0.69	0.19	-0.52	-0.17	0.00	-1.25	0.42	-0.34	-0.79
M_1 - M_3	0.28	-1.31	-1.35	-0.62	0.84	0.36		-1.51	-1.59
M_2 - M_3	-0.46	-1.43	-1,24	-0.63	0.79	1.25		-1.45	-0.95

A table entry represents an outcome of the test statistic Asteriks indicate cases where differences are significant at the 95% level.

Table 9[A] The Ranking of LS, M_1, M_2 and M_3 with the Aid of the Outcomes of the Pitman-Test.

$\rho = 0.0$	b_o	b_1	b_2	$\rho = 0.4$	b_o	b_1	b_2
T = 15	LS M_3 M_2 M_1	LS M_3 M_2 M_1	LS M_3 M_1 M_2	T = 15	M_3 M_1 LS M_2	M_3 LS M_2 M_1	M_3 LS M_1 M_2
T = 25	LS M_3 M_2 M_1	LS M_2 M_3 M_1	LS M_3 M_2 M_1	T = 25	M_3 M_1 LS M_2	M_2 M_3 M_1 LS	M_3 M_1 M_2 LS
T = 40	M_1 M_2 M_3 LS	LS M_2 M_1 M_3		T = 40			M_3 M_1 M_2 LS

Table 9^A

$\rho = 0.8$	b_o	b_1	b_2
T = 15		M_3 M_2 M_1 LS	
T = 25	M_1 M_3 M_2 LS	M_2 M_3 M_1 LS	M_3 M_1 M_2 LS
T = 40		M_2 M_1 M_3 LS	M_1 M_2 M_3 LS

Arrows indicate cases where differences are significant at 95% level.

Table 9^B

The Ranking of LS, M_1, M_2, and M_3 with the Aid of the Outcomes of the Pitman Test.

$\rho = 0.0$	b_o	b_1	b_2	$\rho = 0.4$	b_o	b_1	b_2
T = 15	LS M_3 M_2 M_1	LS M_3 M_1 M_2	LS M_3 M_1 M_2	T = 15	M_3 LS M_2 M_1		M_3 LS M_1 M_2
T = 25	LS M_2 M_3 M_1	LS M_3 M_1 M_2	LS M_3 M_2 M_1	T = 25		M_3 M_1 LS M_2	M_1 M_1 M_2 LS
T = 40	M_2 M_1 LS M_3	LS M_2 M_1 M_3		T = 40		M_3 M_1 M_2 LS	M_3 M_1 M_2 LS

Table 9^B.

$\rho = 0.8$	b_o	b_1	b_2
T = 15		M_3 M_1 M_2 LS	M_3 M_1 M_2 LS
T = 25	M_1 M_3 M_2 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS
T = 40		M_1 M_2 M_3 LS	M_1 M_2 M_3 LS

Arrows indicate cases where differences are significant at the 95% level.

Table 10[A] Results of Pairwise Comparisons with the Aid of the Summers Test.

	$\rho = 0.0$				$\rho = 0.4$				$\rho = 0.8$			
T = 15	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ
LS - M_1	-2.20+	-0.80	-2.20+	-10.00+	-0.00	0.80	0.60	7.00+	3.80+	1.00	2.20+	9.60+
LS - M_2	-2.60+	0.20	-2.80+	-10.00+	-0.20	1.00	1.00	6.80+	2.00+	1.40	2.00+	9.60+
LS - M_3	-4.00+	0.00	0.20	-10.00+	0.80	1.00	1.60	9.00+	3.20+	1.20	2.60+	9.80+
M_1 - M_2	-0.20	1.00	0.40	-1.00	-1.20	0.20	0.80	2.40+	1.60	0.60	0.20	2.80+
M_1 - M_3	-0.80	-1.00	-0.60	-3.00+	0.60	1.20	1.60	1.00	0.60	-0.20	2.00+	6.20+
M_2 - M_3	-0.60	-3.20+	0.80	-2.20+	1.40	0.20	2.00+	0.80	-0.60	-0.20	2.60+	3.20
T = 25	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ
LS - M_1	-0.20	-1.40	-1.00	-10.00+	1.80	1.00	2.20+	8.40+	2.80+	3.00+	2.40+	10.00+
LS - M_3	-0.80	-1.60	-0.80	-10.00+	1.60	0.80	2.20+	8.00	2.60+	3.00+	2.20+	10.00+
LS - M_3	-1.00	-1.40	0.00	-10.00+	2.00	0.60	2.20+	9.60+	2.60+	3.20+	2.20+	10.00+
M_1 - M_2	-0.20	-0.20	-0.40	0.00	-0.40	0.20	0.20	-2.00+	-1.00	0.40	0.30	1.40
M_1 - M_3	-1.20	-0.80	-0.80	-2.40+	-0.80	0.20	0.20	1.00	-2.60+	-0.80	0.60	5.40+
M_2 - M_3	-3.20+	-0.60	-0.80	-2.00+	1.20	1.20	1.00	3.00+	-1.20	-2.20+	1.00	3.80+
T = 40	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ
LS - M_1	0.20	0.60	0.00	-10.00+	1.60	1.60	1.60	9.40+	6.60+	2.40+	5.40+	10.00+
LS - M_2	0.40	0.40	0.00	-10.00+	1.60	1.40	1.60	9.20+	6.20+	2.60+	5.40+	10.00+
LS - M_3	-0.60	-0.40	-1.20	-10.00+	1.80	1.60	1.60	10.00+	6.20+	2.40+	5.40+	10.00+
M_1 - M_2	-0.60	0.60	0.20	1.40	-2.20+	0.40	-2.40+	-0.80	0.20	0.80	-0.60	2.40+
M_1 - M_3	-1.40	-0.60	-1.20	-1.40	-0.40	-0.80	-0.20	2.60+	0.60	-0.40	-1.20	2.80+
M_2 - M_3	-1.20	-0.40	-1.00	-1.40	0.60	-0.20	0.80	3.20+	-0.20	-1.80	-1.20	2.40+

Asteriks indicate cases where differences are significant at the 95% level.

Table 10^{B}. Results of Pairwise Comparisons with the Aid of the Summers Test.

	$\rho = 0.0$				$\rho = 0.4$				$\rho = 0.8$			
T = 15	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ
LS - M_1	-1.00	-1.60	-2.20^{+}	-10.00^{+}	0.80	1.20	0.60	7.00^{+}	0.00	1.80	2.20^{+}	9.60^{+}
LS - M_2	0.00	-1.40	-3.60^{+}	-10.00^{+}	0.60	1.20	1.00	6.80^{+}	-0.20	1.60	2.00^{+}	9.60^{+}
LS - M_3	0.40	-1.80	0.20	-10.00^{+}	1.00	0.60	1.60	9.00^{+}	0.80	1.60	2.60^{+}	9.80^{+}
M_1 - M_2	1.00	0.00	0.20	-1.00	0.60	1.20	0.80	2.40^{+}	-2.20	-0.60	0.20	-2.80^{+}
M_1 - M_3	-0.60	-1.00	-0.60	-3.00^{+}	1.40	1.00	1.60	1.00	0.40	1.00	2.00^{+}	6.20^{+}
M_2 - M_3	-2.20^{+}	0.20	0.80	-2.20^{+}	0.20	1.00	2.00^{+}	0.80	0.80	1.60	2.60^{+}	3.20^{+}
T = 25	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ
LS - M_1	-0.80	-0.40	-1.00	-10.00^{+}	0.80	0.60	2.20^{+}	8.40^{+}	3.00^{+}	1.40	2.40^{+}	10.00^{+}
LS - M_2	-0.20	0.00	-0.80	-10.00^{+}	0.80	0.40	2.20^{+}	9.00^{+}	3.20	1.20	2.20^{+}	10.00^{+}
LS - M_3	-1.40	-0.60	0.00	-10.00^{+}	1.20	0.80	2.20^{+}	9.60^{+}	2.60^{+}	1.20	2.20^{+}	10.00^{+}
M_1 - M_2	0.20	-0.60	-0.40	0.00	0.00	-1.00	-0.20	-2.00^{+}	-1.00	0.20	0.20	1.40
M_1 - M_3	-1.40	-0.80	-0.80	-2.40^{+}	-0.40	0.20	0.20	1.00	-2.20^{+}	0.20	0.60	5.40^{+}
M_2 - M_3	-1.60	-0.20	-0.80	-2.00^{+}	0.60	0.20	1.00	3.00^{+}	-1.20	0.40	1.00	3.80^{+}
T = 40	b_p	b_1	b_2	ρ	b_o	b_1	b_2	ρ	b_o	b_1	b_2	ρ
LS - M_1	0.20	-1.00	0.00	-10.00^{+}	1.60	0.80	1.60	9.40^{+}	3.80^{+}	6.00^{+}	4.50^{+}	10.00^{+}
LS - M_2	0.00	-1.20	0.00	-10.00^{+}	1.40	0.80	1.60	9.20^{+}	3.60^{+}	6.00^{+}	5.40^{+}	10.00^{+}
LS - M_3	-0.20	-1.40	-1.20	-10.00^{+}	1.80	1.40	1.60	10.00^{+}	3.80^{+}	6.00^{+}	5.40^{+}	10.00^{+}
M_1 - M_2	1.20	-0.20	0.00	1.40	0.20	-0.60	-2.20^{+}	-0.80	-0.60	0.80	-0.60	2.40^{+}
M_1 - M_3	-1.00	-1.60	-1.20	-1.40	0.20	0.80	-0.20	2.60^{+}	-1.00	-0.40	-1.20	2.80^{+}
M_2 - M_3	-0.80	-1.40	-1.00	-1.40	0.20	0.20	0.80	3.20^{+}	-0.60	-0.60	-1.20	2.40^{+}

Asterisks indicate cases where differences are significant at the 95% level

Table 11[A].

The Ranking of LS, M_1, M_2, and M_3 with the Aid of the Outcomes of the Summers Test.

$\rho = 0.0$	b_o	b_1	b_2	ρ	$\rho = 0.4$	b_o	b_1	b_2	ρ
T = 15	LS M_1 M_2 M_3			LS M_1 M_2 M_3	T = 15	M_3 LS M_1 M_2	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS
T = 25	LS M_1 M_2 M_3	LS M_1 M_2 M_3		LS M_1 M_2 M_3	T = 25	M_3 M_1 M_2 LS	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS	M_3 M_1 M_2 LS
T = 40	M_1 M_2 LS M_3	M_2 M_1 LS M_3		LS M_2 M_1 M_3	T = 40	M_1 M_3 M_2 LS	M_2 M_1 M_3 LS	M_1 M_3 M_2 LS	M_3 M_1 M_2 LS

Table 11[A].

$\rho = 0.08$	b_o	b_1	b_2	ρ
T = 15	M_2 M_3 M_1 LS	M_2 M_1 M_3 Ls	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS
T = 25	M_1 M_2 M_3 LS	M_2 M_1 M_3 LS	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS
T = 40	M_2 M_3 M_1 LS	M_2 M_1 M_3 LS	M_1 M_2 M_3 LS	M_3 M_2 M_1 LS

Arrows indicate cases where differences are significant at 95% level.

Table 11^B.

The Ranking of LS, M_1, M_2, and M_3 with the Aid of the Outcomes of the Summers Test.

$\rho = 0.0$	b_o	b_1	b_2	ρ	$\rho = 0.4$	b_o	b_1	b_2	ρ
T = 15				LS M_1 M_2 M_3	T = 15	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS	M_3 M_2 M_1 LS
T = 25	LS M_2 M_1 M_3			LS M_1 M_2 M_3	T = 25		M_3 M_1 M_2 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS
T = 40		LS M_1 M_2 M_3		LS M_2 M_1 M_3	T = 40	M_3 M_2 M_1 LS	M_3 M_1 M_2 LS	M_1 M_3 M_2 LS	M_3 M_1 M_2 LS

Table 11[B]. The Ranking of LS, M_1, M_2, and M_3 with the Aid of the Outcomes of the Summers Test

$\rho = 0.8$	b_o	b_1	b_2	ρ
T = 15	M_3 LS M_1 M_2	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS
T = 25	M_1 M_2 M_3 LS	M_3 M_1 M_2 LS	M_3 M_1 M_2 LS	M_3 M_2 M_1 LS
T = 40	M_1 M_2 M_3 LS	M_2 M_1 M_3 LS	M_1 M_2 M_3 LS	M_3 M_2 M_1 LS

Arrows indicate cases where differences are significant at 95% level.

Table 12^A. Results of Applying the X^2 test 10df. to the Sample Distributions of the Studentized coefficients b_o, b_1, and b_2.

		T = 15			T = 25			T = 40		
		b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
LS	ρ = 0.0	6.3	9.9	10.6	7.0	8.3	9.3	5.3	14.3	15.9
	ρ = 0.4	8.5	11.1	8.2	11.4	9.7	9.5	5.2	16.5	18.2
	ρ = 0.8	7.1	28.1^+	15.7	5.7	9.9	15.5	24.4^+	17.2	17.0
M_1	ρ = 0.0	14.3	13.5	8.2	9.1	10.7	13.4	9.8	5.8	10.8
	ρ = 0.4	4.2	6.5	8.7	12.1	6.4	7.8	8.8	17.9	8.3
	ρ = 0.8	10.0	6.0	18.5^+	8.8	6.6	6.2	18.8^+	4.0	5.6
M_2	ρ = 0.0	12.5	16.4	7.2	8.2	13.3	12.6	11.3	11.1	12.7
	ρ = 0.4	3.4	10.8	13.8	14.7	7.8	8.2	7.1	16.7	13.8
	ρ = 0.8	19.7^+	5.6	15.6	6.2	6.1	4.0	17.3	5.1	5.7
M_3	ρ = 0.0	14.0	19.7^+	8.7	11.0	12.1	15.0	8.2	6.2	9.6
	ρ = 0.4	6.4	9.3	3.3	10.8	7.2	10.3	8.0	10.5	8.7
	ρ = 0.8	19.1^+	6.2	15.8	11.6	3.7	5.6	17.9	2.5	3.7

Asterisks indicate cases where differences are significant at 95% level.

Table 12^B.

Results of Applying the X^2 Test 10df. to the Sample Distributions of the Studentized coefficients b_o, b_1 and b_2.

		T = 15			T = 25			T = 40		
		b_o	b_1	b_2	b_o	b_1	b_2	b_o	b_1	b_2
LS	ρ = 0.0	6.6	9.1	12.8	13.8	6.4	9.3	8.5	17.9	16.3
	ρ = 0.4	10.2	6.0	6.8	37.2^+	8.0	10.0	15.4	7.1	17.5
	ρ = 0.8	8.3	4.8	17.5	12.6	23.4^+	15.4	6.4	10.3	17.0
M_1	ρ = 0.0	22.3^+	7.4	10.0	22.0^+	12.8	14.0	9.1	27.6^+	12.6
	ρ = 0.4	7.7	7.3	9.4	18.8^+	15.6	8.6	2.8	9.5	8.3
	ρ = 0.8	11.4	14.3	18.5^+	2.7	9.4	6.2	9.4	5.0	5.6
M_2	ρ = 0.0	18.4^+	8.7	7.2	19.7^+	12.2	13.5	6.1	19.5^+	11.6
	ρ = 0.4	5.8	8.5	15.5	17.6	12.0	11.7	6.4	7.7	9.0
	ρ = 0.8	13.2	21.0^+	15.4	1.6	12.9	3.4	12.8	3.6	5.7
M_3	ρ = 0.0	14.2	11.0	8.7	17.6	18.8^+	15.0	6.0	15.3	9.6
	ρ = 0.4	7.6	5.1	3.3	23.6^+	14.5	3.2	2.6	10.6	8.7
	ρ = 0.8	3.6	19.1^+	18.7	8.5	13.4	5.6	8.3	5.6	3.7

Asterisks indicate cases where differences are significant at 95% level.

Table 13. Average Scores for the Ranking with the Aid of the Pitman Test.

MODEL A

$\rho = 0.0$	T = 15	T = 25	T = 40
LS	17	$15\frac{1}{2}$	$1\frac{1}{2}$
M_1	2	0	$2\frac{1}{2}$
M_2	$2\frac{2}{3}$	4	$2\frac{1}{2}$
M_3	12	$3\frac{1}{3}$	1
$\rho = 0.4$	T = 15	T = 25	T = 40
LS	3	2	0
M_1	$1\frac{1}{2}$	8	9
M_2	$\frac{1}{2}$	$3\frac{1}{3}$	6
M_3	$14\frac{2}{3}$	$9\frac{2}{3}$	10
$\rho = 0.8$	T = 15	T = 25	T = 40
LS	0	0	0
M_1	6	$10\frac{2}{3}$	10
M_2	12	8	11
M_3	18	11	6

MODEL B

$\rho = 0.0$	T = 15	T = 25	T = 40
LS	17	16	5
M_1	4	$\frac{1}{3}$	$2\frac{1}{2}$
M_2	2	3	4
M_3	$10\frac{2}{3}$	$3\frac{1}{3}$	0
$\rho = 0.4$	T = 15	T = 25	T = 40
LS	6	$\frac{1}{2}$	0
M_1	1	8	8
M_2	1	1	6
M_3	$14\frac{1}{2}$	$9\frac{1}{2}$	10
$\rho = 0.8$	T = 15	T = 25	T = 40
LS	0	0	0
M_1	8	$12\frac{2}{3}$	$11\frac{1}{2}$
M_2	$3\frac{1}{2}$	6	$8\frac{1}{2}$
M_3	17	$12\frac{2}{3}$	6

Table 14. Average scores for the Ranking with the Aid of the Summers test.

MODEL A				MODEL B			
ρ = 0.0	T = 15	T = 25	T = 40	ρ = 0.0	T = 15	T = 25	T = 40
LS	18	11	7	LS	18	12	$10\frac{2}{3}$
M_1	6	$4\frac{1}{3}$	$4\frac{1}{3}$	M_1	9	5	4
M_2	4	$4\frac{2}{3}$	$4\frac{2}{3}$	M_2	6	$5\frac{1}{2}$	4
M_3	0	0	0	M_3	0	0	0
ρ = 0.4	T = 15	T = 25	T = 40	ρ = 0.4	T = 15	T = 25	T = 40
LS	1/2	0	0	LS	0	0	0
M_1	$3\frac{1}{4}$	9	$8\frac{3}{4}$	M_1	$3\frac{1}{4}$	8	$6\frac{3}{4}$
M_2	$5\frac{1}{4}$	$6\frac{1}{2}$	$4\frac{1}{2}$	M_2	$6\frac{3}{4}$	$4\frac{1}{3}$	4
M_3	$9\frac{3}{4}$	$12\frac{1}{2}$	$8\frac{1}{2}$	M_3	10	$9\frac{2}{3}$	9
ρ = 0.8	T = 15	T = 25	T = 40	ρ = 0.8	T = 15	T = 25	T = 40
LS	0	0	0	LS	1/2	2	0
M_1	$6\frac{1}{4}$	$8\frac{3}{4}$	$7\frac{1}{2}$	M_1	5	$9\frac{1}{3}$	$8\frac{3}{4}$
M_2	$9\frac{1}{4}$	$9\frac{1}{2}$	$10\frac{1}{4}$	M_2	$5\frac{1}{2}$	8	$9\frac{3}{4}$
M_3	$12\frac{1}{2}$	$10\frac{1}{4}$	$9\frac{1}{2}$	M_3	17	$11\frac{2}{3}$	9

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